1 Topics: Elementary set theory, the sample space, counting

1.1 Prerequisites: Lecture 1

Exercise 1- 1: (Suggested for personal/peer tutorial) Let A, B and C be three arbitrary events. Which of the following relationships are true? Justify your answers.

(a) $(A \cup B) \cap (A \cup C) = A \cup (B \cap C)$.

(b) $(A \cup B) = (A \cap B^c) \cup B$.

(c) $(A^c \cap B) \cup (A \cap B^c) = (A \cup B) \cap (A \cap B)^c$.

(d) $(A \cup B)^c \cap C = A^c \cap B^c \cap C^c$.

(e) $(A \cap B) \cap (B^c \cap C) = \emptyset$.

Exercise 1- 2: A coin is tossed three times. Let A be the event that there are exactly two heads, B the event that there are more heads than tails, and C the event that the last toss is a tail. Using the operations of union, intersection and complement, find in terms of A, B and C expressions for the events:

(a) There are more tails than heads.

(b) There are three heads.

(c) The first two tosses are heads.

Exercise 1- 3: A computer hardware company manufactures five apparently identical terminals, two of which are actually defective. An order for two terminals is received, and is filled by selecting two of the five.

(a) List the elements of the sample space corresponding to how the order is filled.

(b) Let A denote the event that the order is filled with two non-defective terminals. List the sample points in A.

1.2 Prerequisites: Lecture 2

Exercise 1- 4: Consider a horse race with 10 horses. We assume that all 10 horses will complete the race and that there are no ties. How many combinations are there for the first, second and third place winners?

Exercise 1- 5: Consider buying one scoop of ice cream. You can choose either a cup or a cone and there are three possible flavours: chocolate (C), vanilla (V) and strawberry (S). Draw two possible trees to visualise the number of possibilities.

1.3 Prerequisites: Lecture 3

Exercise 1- 6: Revisit Exercise 1- 4 and describe how you can solve it using the idea of sampling without replacement.

Exercise 1- 7: Suppose we roll two fair dice and we would like to determine the probabilities of a sum of 11 and a sum of 12. Leibniz argued that both events are equally likely since each outcome can only be obtained in one way. Do you agree with Leibniz? Justify your answer!