Topic: Continuous random variables and their distributions

In today's problem class we will be studying properties of continuous random variables.

1. For each of the functions $f(x)$ given below determine whether $f(x)$ is a valid probability density function (p.d.f.). If $f(x)$ is not a valid p.d.f., determine if there exists a constant c such that $cf(x)$ is a valid p.d.f.. Note that in each case, $f(x) = 0$ for all x not in the interval specified.
   (a) $f(x) = 2x, \quad 0 < x < 1.$
   (b) $f(x) = |x|, \quad |x| < \frac{1}{2}.$
   (c) $f(x) = 1 - |x|, \quad |x| < 1.$
   (d) $f(x) = \log(x), \quad 0 < x < 1.$
   (e) $f(x) = \log(x), \quad 0 < x < 2.$
   (f) $f(x) = \frac{2}{3}(x-1), \quad 0 < x < 3.$
   (g) $f(x) = e^{-2x}, \quad x > 0.$
   (h) $f(x) = 4e^{-2x} - e^{-x}, \quad x > 0.$
   (i) $f(x) = e^{-|x|}, \quad |x| < 1.$

2. Let $Z \sim \mathrm{N}(0,1)$. Let $\mu \in \mathbb{R}$ and $\sigma > 0$. Find the c.d.f. and the p.d.f. of the random variable $X = \sigma Z + \mu$. Note that you can express the c.d.f. of X in terms of the c.d.f. Φ of Z.

3. You are bidding against a competitor for an item on eBay. The amount, X, in pounds, of the bid placed by your competitor has probability density function given by:

$$f_X(x) = \begin{cases} c(20-x), & 0 < x < 20; \\ 0, & \text{otherwise.} \end{cases}$$

   You make a bid without knowing your competitor's bid.
   (a) Determine the value of c.
   (b) Find $F_X(x)$, the cumulative distribution function (cdf) of X.
   (c) What is the probability that you lose the bid if you place a bid of £16?
   (d) How much should you bid in order to have a 75% chance of winning?