Exercise 7.1. Consider a discrete metric space $(X, \mathrm{d}_{\mathrm{disc}})$, that is $\mathrm{d}_{\mathrm{disc}}$ is a discrete metric on X. Show that $\mathrm{d}_{\mathrm{disc}}$ induces the discrete topology on X.

Exercise 7.2. Let (X, τ) be a topological space, $Y \subset X$, and

$$\tau_Y = \{U \cap Y \mid U \in \tau_X\}.$$

Show that τ_Y is a topology on Y.

Exercise 7.3. Let τ_{Eucl} be the Euclidean topology on $\mathbb{R}$, that is τ_{Eucl} is the collection of all open sets in $(\mathbb{R}, \mathrm{d}_1)$. Show that the collection

$$\{U \times V \mid U \in \tau_{\mathrm{Eucl}}, V \in \tau_{\mathrm{Eucl}}\}.$$

is not a topology on $\mathbb{R} \times \mathbb{R}$. Is condition T2 satisfied? How about condition T3?

Exercise 7.4. Let (A, τ) be a topological space, and let S and T be subsets of A. The following properties hold:

(i) if $S \subset T$ then $S^\circ \subset T^\circ$,

(ii) S is open in A if and only if $S = S^\circ$,

(iii)* S° is the largest open set contained in S.

Exercise 7.5. Let (X, d) be a metric space, and let τ be the topology on X induced from the metric d. Show that (X, τ) is a Hausdorff topological space.

Exercise 7.6. Assume that the topological spaces (X, τ_X) and (Y, τ_Y) are topologically equivalent. Then, (X, τ_X) is Hausdorff if and only if (Y, τ_Y) is Hausdorff.

Please send any corrections to d.cheraghi@imperial.ac.uk
Questions marked with $*$ are optional