Groups and Rings: Unseen Problem Sheet 1

hannah.tillmann-morris15@ic.ac.uk

October 2021

Question 1. Let G and H be groups. Denote by $\mathrm{Hom}(G, H)$ the group of homomorphisms from G to H (the operation being the group operation in H). Describe the following groups:

1. $\mathrm{Hom}(\mathbb{Q}, \mathbb{Z})$
2. $\mathrm{Hom}(\mathbb{Z}/n\mathbb{Z}, \mathbb{Z})$
3. $\mathrm{Hom}(\mathbb{Z}/n\mathbb{Z}, \mathbb{C}^{\times})$

In the above, you should consider $\mathbb{Q}$ and $\mathbb{Z}$ with the operation being addition. $\mathbb{C}^{\times}$ has multiplication as its operation.

Question 2. Let H be a normal subgroup of G and K be a normal subgroup of H. Is K a normal subgroup of G?

Question 3. Let H be a subgroup of G such that $g^2 \in H$ for any $g \in G$.
Prove H is a normal subgroup of G.

Question 4. Prove that an infinite group G has infinitely many subgroups.

Question 5. Prove the following statement or find a counterexample.
Let H and K be subgroups of a group G and L be a normal subgroup of G. If $HL = KL$ and $H \cap L = K \cap L$, then $H = K$.