

# Probability for Statistics

## Unseen Problem 4

1. Suppose that  $X$  and  $Y$  are absolutely continuous random variables with joint pdf given by

$$f_{X,Y}(x,y) = cx(1-y), \text{ for } 0 < x < 1 \text{ and } 0 < y < 1,$$

and zero otherwise, for some constant  $c$ .

- (a) Are  $X$  and  $Y$  independent random variables?
  - (b) Find the value of  $c$ .
  - (c) Find  $\Pr(X < Y)$ .
2. Let  $X$  be a  $2 \times 2$  symmetric matrix with random entries. Suppose  $X_{11}, X_{22} \sim N(0, 1)$  and  $X_{12} \sim N(0, \frac{1}{2})$ , with all mutually independent. Let the eigenvalues of  $X$  be  $\lambda_1$  and  $\lambda_2$ . Find the distribution of the eigenvalue spacing  $|\lambda_1 - \lambda_2|$ .