GALOIS THEORY
Worksheet 5

Q 1. Prove that $\mathbb{Q} \subset \mathbb{Q}(\sqrt{2}, \sqrt{-3}, \sqrt[3]{5})$ is a normal extension. What is its degree?

Q 2. (a) Prove that if $K \subset L$ is a field extension and $[L : K] = 2$ then the extension is normal.

(b) Prove that every index 2 subgroup of a group G is normal.

Q 3. Say $r = \sqrt[11]{5^{1/3} + \sqrt{8^{1/5} + 6}} + 9^{1/7}$. Find a sequence of fields $\mathbb{Q} = F_0 \subseteq F_1 \subseteq F_2 \subseteq \cdots \subseteq F_n$ with $r \in F_n$ and such that for all i we have $F_i = F_{i-1}(\alpha_i)$ with $\alpha_i^{n_i} \in F_{i-1}$ for some positive integer n_i.

Q 4. Say K is the splitting field of $X^3 - 11$ over $\mathbb{Q}$. Figure out the Galois group of the extension $\mathbb{Q} \subset K$. List all the subfields of K, all the subgroups of the Galois group, and draw a picture of the Galois correspondence.