Problem Sheet 3

We will discuss the solutions of this problem sheet in the problem class on Friday, 17 February 2023.

1. Suppose you observe a firm using two input goods with prices (w_1, w_2) and one output good with price p. You have

t	p	w_1	w_2	x_1	x_2	y
1	2	2	1	10	20	100
2	3	1	2	14	10	110

Check whether the Weak Axiom of Profit Maximisation is satisfied and check whether the Weak Axiom of Cost Minimisation is satisfied.

2. Suppose the two input goods have prices $w_1 > 0$ and $w_2 > 0$. In the lecture we have seen that a necessary first-order condition for the cost minimisation problem for some fixed output $y > 0$ is given by

$$\mathrm{MRTS}(x_1^*, x_2^*) = -\frac{w_1}{w_2} \tag{1}$$

for the cost minimising input bundle $\underline{x}^* = (x_1^*, x_2^*) \in \mathbb{R}^2_{\geq 0}$ if the production function is differentiable. Graphically, the general situation can be illustrated as in figure 1 for a firm with increasing, continuous and quasi-concave production function $f\colon \mathbb{R}^2_{\geq 0} \to \mathbb{R}$.

a) Why is the cost minimising consumption bundle $\underline{x}^*$ necessarily on the iso-quant $f^{-1}(\{y\})$?

b) What does a line with slope $-w_1/w_2$ and intercept $K \geq 0$ represent in this context economically?

c) Determine the value of $-w_1/w_2$ using Figure 1.

d) What does condition (1) mean graphically?

e) Determine the cost minimising input bundle $\underline{x}^* = (x_1^*, x_2^*)$ in figure 1. What can you say about the total costs when using $\underline{x}^*$?

f) Suppose that the price for good 1 increases to $w_1' > w_1$ whereas the price for good 2 remains constant. Determine the new cost minimising input bundle graphically using figure 1.

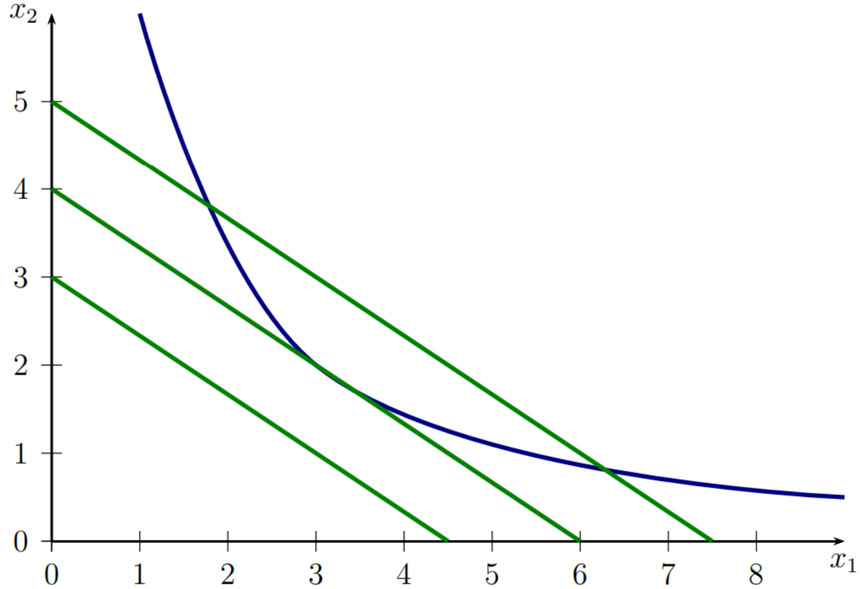

Figure 1: Several lines (green) with slope $-w_1/w_2$ and different intercepts. The curve (blue) is the isoquant $f^{-1}(\{y\})$.

g) Now consider a Leontief production function $f(x_1, x_2) = \min(x_1, x_2)$. Determine the cost minimising input bundle graphically, similarly to the approach in figure 1. To this end, consider the following situations.

- $y = 2$ and $w_1 = w_2 = 1$.
- $y = 2$, $w_1 = 2$ and $w_2 = 1$.

Use the same graph. (Label the axes adequately.)

3. Consider a firm with three inputs. The first two inputs are used for the actual production with a Cobb-Douglas technology whereas the third one limits the maximal amount of output. Moreover, assume that the first two input goods are variable in the short run, but the third one is fixed in the short run and only variable in the long run. Therefore, the production function takes the form:

$$f : \mathbb{R}^3_{\geq 0} \to \mathbb{R}_{\geq 0}, \qquad f(x_1, x_2, x_3) = \min\{x_1^{1/3} x_2^{2/3}, x_3\}$$

a) Think of an example where such a production function might occur (also with the fixed and variable costs).

b) Does f satisfy the two conditions we imposed on production functions (monotonicity and quasi-concavity)? What is the behaviour of f with respect to scale?

c) Compute the cost function and the short run cost function.

d) Suppose that the prices w_1, w_2, w_3 are such that

$$c^*(y) = 8y, \qquad c_s^*(x_3, y) = 6y + 2x_3,$$

where c_s^* is only defined for $x_3 \geq y$. Can you explain why this is the case?

e) Using the cost functions from **d)** describe the optimal short-run and long-run behaviour of the firm, if the output price is $p = 5$, $p = 7$, and $p = 10$, and if the fixed input is initially at $x_3 = 10$.

f) Now, determine the optimal short-run and long-run behaviour for the firm if the prices are given by the inverse demand function $p(y) = 10 - y$ for $0 \leq y \leq 10$ (again, assuming that $x_3 = 10$ initially).