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1. [default,M2]

In the framework of the N-period binomial model with constant parameters $S_0 = 8, u = 2, d = 1/2, r = 0$, let $S = (S_n)_{n=0}^N$ be the stock price process, M_n its historical *minimum* up to time n (i.e. $M_n := \min_{i=0,\dots,n} S_i$). Consider the down-and-in rebate option with the lower barrier $L = 6$ which expires at time N and pays 1 if S_n is less than L for any $n = 0, \dots, N$; in other words, this derivative has a payoff $1 - Y_N$ at maturity N, where $Y_n := 1_{\{M_n \geq 6\}}$ (i.e. $Y_n = 1$ if $M_n \geq 6$, and $Y_n = 0$ otherwise). We denote with V_n the arbitrage-free price at time $n = 0, \dots, N$ of this option.

Below, whenever we say that a process is Markov, we mean with respect to the risk-neutral measure $\mathbb{Q}$ and with the usual filtration $\mathcal{F}_n = \sigma(X_1, \dots, X_n), n = 0, \dots, N$ generated by the coin tosses $X_n(\omega) = \omega_n$ on the probability space $\Omega = \{H, T\}^N$. Answer the following questions and justify carefully with either proofs or counterexamples.

(a) Draw the binary tree representing S. Can you draw it as a recombinant tree?

A. No B. Yes

(b) Are $(X_n)_{n \leq N}$ independent under $\mathbb{Q}$?

A. No B. Yes

(c) Are $(X_n)_{n \leq N}$ identically distributed under $\mathbb{Q}$?

A. No B. Yes

(d) Compute $\mathbb{Q}(\{\omega\})$ for every $\omega \in \{H, T\}^N$, then choose the correct statement

A. $\mathbb{Q}(\{\omega\})$ is constant in $\omega \in \{H, T\}^N$

B. $\mathbb{Q}(\{\omega\})$ is not constant, but depends only on the number of heads in $\omega \in \{H, T\}^N$

C. None of the above

(e) Is S Markov?

A. No B. Yes

(f) Is M a Markov? A. No B. Yes

(g) Is Y a Markov? A. No B. Yes

(h) Is (S, M) a Markov process? A. No B. Yes

(i) Is (S, Y) a Markov process? A. No B. Yes

(j) Is (S, Y, M) a Markov process? A. No B. Yes

(k) Which of the above processes $W = (W_n)_n$ are Markov and are such that, for every $n = 0, \dots, N$, V_n admits the representation $V_n = v_n(W_n)$, for some Borel $v_n : \mathbb{R} \to \mathbb{R}$?

A. S B. M C. Y D. (S, M) E. (S, Y) F. (S, Y, M)

(l) Among several the choices of W which you selected in the previous question, which one would be best (i.e. lead to the shortest computations) to use to compute price explicitly

A. S B. M C. Y D. (S, M) E. (S, Y) F. (S, Y, M)