This document contains 2 questions.

1. [default,Q6]

   An *exchange option* with maturity $T \geq 0$ gives its holder the right (but not the obligation!) to receive one unit of S^1 and give in return one unit of S^2 at time T, where S^1 and S^2 are stocks in two different companies. Suppose today a hedge-fund is willing to trade with you an exchange option with expiration in 3 months for 2 and the prices of the stocks are $S_0^1 = 100, S_0^2 = 97$. Can you make an arbitrage?

   A. Yes: buy the option, etc. B. Yes: sell the option, etc. C. No D. Not enough info to answer

   In the answers, by 'etc' I mean that you can additionally trade in S^1, S^2, and the bank account.

2. [default,O13]

   In a market composed of one underlying, and of a bank account with interest rate r between now and time T, consider the following derivatives:

   1. Option F gives its buyer the right, and the obligation, to buy at expiry $T > 0$ the underlying at a price $K \in \mathbb{R}$ (fixed in advance), instead of at its market price P_T (so, the buyer of the option will have to buy the underlying at a price K, under all possible circumstances). Thus, the final value (payoff) of option F is $F_T = P_T - K$. Notice that, if the value of K is set equal to the T-forward price $P_0(1 + r)$, where P_0 is the initial price of the underlying, then option F is the forward contract and its initial value (price) is $F_0 = 0$.
   2. Option A *(resp. B)* gives its buyer the right, but not the obligation, to buy *(resp. to sell)*, at expiry T, the underlying at a price K (so, at expiry the buyer can choose whether buy the underlying at a price K, or not). Fyi, option A is called a *call* option, and option B a *put* option.
   3. Option C *(resp. D)* gives its buyer the obligation, but not the right, to buy *(resp. to sell)*, at expiry T, the underlying at a price K (so, at expiry the seller of the option can choose whether its buyer should buy the underlying at a price K, or not, and the buyer will be obligated to do what the seller decided).

   Notice that the value of $K \in \mathbb{R}$ (which is known as the *strike price*) is arbitrary, and so the initial prices of these options are in general not 0. Call X_t the value of option (X) (for $X \in \{A, B, C, D\}$) at time $t \in \{0, T\}$. We will use the following standard notation: by definition, the positive (*negative*) part of $x \in \mathbb{R}$ is $x^+ := \max(x, 0)$ and $x^- := -\min(x, 0)$ (the minus sign in from the *min* is not a typo!). Notice that $x^- = (-x)^+$ and $x = x^+ - x^-$ for all $x \in \mathbb{R}$.

   (a) Write the payoffs of the above options as functions of the underlying:

      i. What is the payoff of option A ?
      A. $P_T - K$ B. $(K - P_T)^+$ C. $(P_T - K)^+$ D. $-(P_T - K)^-$

      ii. What is the payoff of option B ?
      A. $P_T - K$ B. $(K - P_T)^+$ C. $(P_T - K)^+$ D. $-(K - P_T)^-$

      iii. What is the payoff of option C ?
      A. $K - P_T$ B. $(K - P_T)^+$ C. $-(P_T - K)^-$ D. $-(K - P_T)^-$

      iv. What is the payoff of option D ?
      A. $(P_T - K)^+$ B. $(K - P_T)^+$ C. $-(P_T - K)^-$ D. $-(K - P_T)^-$

(b) For what value of $(a, b) \in \mathbb{R}^2$ do the payoffs of options A, B, F satisfy the linear relation $aA_T + bB_T = F_T$?

A. $(1, -1)$ B. $(1, 1)$ C. $(-1, 1)$ D. $(1, 0)$

(c) If K equals the T-forward price of the underlying, and the put option has price B_0 at time 0, what is the price A_0 of the call option?

A. 0 B. B_0 C. $B_0 + P_0$ D. $B_0/(1+r)$

(d) Using your intuition, can you tell which of the prices A_0, B_0, C_0, D_0 should always be positive (i.e. ≥ 0), and which should always be negative (i.e. ≤ 0)?

A. $A_0 \geq 0,\ B_0 \leq 0,\ C_0 \geq 0,\ D_0 \leq 0$
B. $A_0 \geq 0,\ B_0 \geq 0,\ C_0 \geq 0,\ D_0 \geq 0$
C. $A_0 \leq 0,\ B_0 \leq 0,\ C_0 \geq 0,\ D_0 \geq 0$
D. $A_0 \geq 0,\ B_0 \geq 0,\ C_0 \leq 0,\ D_0 \leq 0$

(e) Consider the *strangle* option, whose payoff equals that of a put option with strike price K_1 plus that of a call option with strike price $K_2 > K_1$ (both options have the same underlying S and expiry T). For what function f is the strangle's payoff $= f(S_T)$?

A.
$$f(x) = \begin{cases} x - K_2 & \text{if } x > K_2 \\ 2x - K_1 - K_2 & \text{if } K_1 \leq x \leq K_2 \\ K_1 - x & \text{if } x < K_1 \end{cases}$$

B. $f(x) = K_2 - K_1$

C.
$$f(x) = \begin{cases} x - K_1 & \text{if } x > K_2 \\ 0 & \text{if } K_1 \leq x \leq K_2 \\ K_2 - x & \text{if } x < K_1 \end{cases}$$

D.
$$f(x) = \begin{cases} x - K_2 & \text{if } x > K_2 \\ 0 & \text{if } K_1 \leq x \leq K_2 \\ K_1 - x & \text{if } x < K_1 \end{cases}$$

(f) Complete the following sentence:

If $K_1 < P_0 < K_2$ and I want to buy the above portfolio, it means that I believe that the underlying's price P_T will

A. change quite dramatically in either direction.
B. decrease quite dramatically.
C. increase quite dramatically.
D. not change much.
E. not increase much.
F. not decrease much.