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1. [default,O3a]

On the probability space $\Omega = \{\omega_1, \omega_2, \omega_3\}$, on which is defined probability $\mathbb{P}$ such that $\mathbb{P}(\{\omega\}) > 0$ for every $\omega \in \Omega$, define random variables

ω	ω_1	ω_2	ω_3
$S_1(\omega)$	1	6	12
$X_1(\omega)$	22	30	44
$Y_1(\omega)$	22	32	44

Consider the one-period trinomial model of the market (B, S) made of a bond B with initial price 1 and interest rate $r = 1$, a one stock whose initial price is $S_0 = 3$, and whose final price S_1 is as in the above table. Consider also the derivatives with payoffs X_1, Y_1. Denote with $u(X) := u(X; B, S)$ (resp. $d(X) := d(X; B, S)$) the smallest (resp. largest) value at which an infinitely risk-averse agent, investing in the market (B, S), is willing to sell (resp. buy) X. So far, all prices were stated in a fixed currency, say £. When solving this exercise, compute all values not in terms of £ but in terms of units of bond. In other words, given a process of prices $W = (W_0, W_1)$, consider instead the discounted process

$$\overline{W}_t := W_t/B_t, \text{ i.e. } \overline{W}_0 = W_0, \quad \overline{W}_1 = W_1/(1+r)$$

(so e.g. taking $W = B$ this means $\overline{B}_0 = \overline{B}_1 = 1$, taking $W = S$ this means $\overline{S}_0 = S_0, \overline{S}_1 = S_1/(1+r)$). Recall that a model is called *complete* if any derivative can be replicated in such model.

In item (g), we consider the enlarged market (B, S, Y), where we are assuming that Y_1 is being sold at price $Y_0 := 16$ at time 0. From item (h) (included) onwards, we consider the enlarged market (B, S, X), where we are assuming that X_1 is being sold at price $X_0 := 31/2$ at time 0.

(a) Is the model (B, S) free of arbitrage?
A. No B. Yes

(b) Is the model (B, S) complete?
A. No B. Yes

(c) Is X_1 replicable in the model (B, S)?
A. No B. Yes

(d) Is Y_1 replicable in the model (B, S)?
A. No B. Yes

(e) What are $d(X; B, S), u(X; B, S)$?
A. 15,15 B. 16,16 C. 15,16 D. $\frac{31}{2}, \frac{31}{2}$ E. None of the above

(f) What are $d(Y; B, S), u(Y; B, S)$?
A. 15,15 B. 16,16 C. 15,16 D. $\frac{31}{2}, \frac{31}{2}$ E. None of the above

(g) Is the model (B, S, Y) complete?
A. No B. Yes

(h) Is the model (B, S, X) arbitrage-free?
A. No B. Yes

(i) Is the model (B, S, X) complete?

A. No   B. Yes