This document contains 2 questions.

1. [default,O3g]

   Consider the probability space $\Omega = \{\omega_1, \omega_2, \omega_3\}$, with a probability $\mathbb{P}$ such that $\mathbb{P}(\{\omega\}) > 0$ for every $\omega \in \Omega$. For $t \in \mathbb{R}$, define the random variables

ω	ω_1	ω_2	ω_3
$S_1(\omega)$	2	3	5
$X_1(\omega)$	1	3	t

   Consider the one-period trinomial model of the market (B, S) made of a bond B with initial price 1 (all prices in a fixed currency, say £), and interest rate $r = 0$, a stock whose initial price is $S_0 = 4$, and whose final price is S_1. We also consider a derivative X with payoff X_1. Answer the following questions and justify carefully with either proofs or counterexamples.

   (a) Is the market (B, S) free of arbitrage?

   A. No B. Yes

   (b) Find all the values of t (if any) for which X_1 replicable (in the market (B, S)).

   A. 8 B. 5 C. None of the above

   Assume from now on that X has initial cost $X_0 = 4$.

   (c) Find all the values of t (if any) for which the market (B, S, X) is free of arbitrage.

   A. 5 B. $\frac{11}{2}$ C. $(5, \frac{11}{2})$ D. $[5, \frac{11}{2}]$ E. None of the above

   Assume from now on that $t = 5$.

   (d) Can EMM be used to determine whether the market (B, S, X) is complete?

   A. No B. Yes

   (e) Find explicitly an arbitrage strategy in the market (B, S, X).

2. [default,O16b]

   Consider a one-period arbitrage-free model (B, S) of market composed of a bank account B with interest rate r, and one underlying S. To find the price of a money-back call option, complete the following steps.

   (a) Write down a formula for the function f s.t. $f(S_1, m)$ is the payoff of the derivative which, at maturity $T = 1$, provides its buyer with both a call option with strike K on the underlying S, and the amount $m \in \mathbb{R}$ if the call with strike K is in the money (i.e. if $S_T \geq K$).

   (b) From now on let (B, S) be given by the binomial model with

   $$r = 0, \quad S_0 = 12, \quad S_1(H) = 20, \quad S_1(T) = 4, \quad K = 12.$$

   Compute the initial value $X_0(m)$ of the derivative with payoff $X_1(m) := f(S_1, m)$.

   (c) In the above binomial model (B, S), compute the price M_0 of the *money-back call* option, which at maturity $T = 1$ gives its buyer a call option on S with strike $K = 12$, plus it repays the initial cost M_0 if the call with strike K finished in the money.

   What is the value of M_0?

   A. 10 B. 8 C. 4 D. None of the above