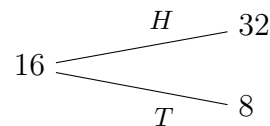


This document contains 2 questions.

1. [default,Q11]

A put option is the contract that gives the buyer the right (but not the obligation) to *sell* a share of the underlying to the seller at maturity at strike price $K > 0$. Consider the (arbitrage-free) one-period binomial model with interest rate $r = \frac{1}{4}$ and underlying S given by $S_0 = 16$, $S_1(H) = 32$, $S_1(T) = 8$, i.e. by the tree



If I have to hedge a short position in the put option with strike 16 (and maturity 1):

(a) What is the required initial capital V_0 ?

- A. $\frac{1}{3}$ B. $-\frac{1}{3}$ C. $\frac{16}{5}$ D. $-\frac{16}{5}$ E. None of the above

(b) How much cash $k \in \mathbb{R}$ should I keep in the bank?

- A. $\frac{128}{15}$ B. $-\frac{128}{15}$ C. $\frac{1}{3}$ D. $-\frac{1}{3}$ E. None of the above

(c) How many shares $h \in \mathbb{R}$ of the underlying should I buy?

- A. $\frac{1}{6}$ B. $-\frac{1}{6}$ C. $\frac{1}{3}$ D. $-\frac{1}{3}$ E. None of the above

(d) What is the price of the put (i.e., at what price should the put be sold)?

- A. V_0 B. $-V_0$ C. None of the above

2. [default,O16]

Consider a one-period binomial model with expiry $T = 1$, of a market composed of one underlying S and a bank account with interest rate r . Recall that a call (resp. put) option with strike K is the contract that gives you the right (but not the obligation) to buy (resp. sell) one unit of the underlying at price K at maturity. Assume that $r = 0$, $S_0 = 8$, and S_1 can take the values 4 and 16. Given constants $k = 8$, $k_1 = 6$, $k_2 = 12$, $c = 4$, consider the following derivatives; when a derivative is only specified in words, you first have to write the formula for its payoffs.

(a) A *capped call*, which has payoff function

$$C(x) := \min [(x - k)^+, c],$$

i.e. the payoff of the derivative is $C(S_T)$ at expiry. What is the price of this derivative?

- A. $-4/3$ B. 6 C. $4/3$ D. None of the above

(b) A (long) *range-forward* contract is the portfolio long 1 put of strike k_1 , and short 1 call of strike $k_2 > k_1$; the call and put have the same underlying and expiry. What is the price of this derivative?

- A. $-2/3$ B. 4 C. 0 D. None of the above