This document contains 2 questions.

1. [default,O4]

   Consider a one period market model composed of a bank account with constant interest rate $r = 2$ and a stock whose price at time 0 is $S_0 > 0$. Assume that the stock price S_1 at time one is given by $S_1 = 1 + N$, where N is a Poisson distributed random variable with parameter one.

   (a) For what values of S_0 is the market free of arbitrage?

   A. $S_0 < 1/3$ B. $S_0 \leq 1/3$ C. $S_0 > 1/3$ D. $S_0 \geq 1/3$ E. $S_0 > 1$ F. $S_0 \geq 1$

   (b) When this market is free of arbitrage, is it complete?

   A. No B. Yes C. It depends on the exact value of S_0

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2. [default,O19]

   Consider the one-period binomial model with interest rate $r = 0$ and stock price with value $S_0 = 150$ at time 0, and which takes the values $200, 140$ at time 1; thus, we model its price at time 1 with the random variable $S_1(H) = 200, S_1(T) = 140$, defined on $\Omega := \{H, T\}$. Assume that the probability of $\{S_1 = 200\}$ is $\frac{9}{10}$, i.e. $\mathbb{P}(\{H\}) = \frac{9}{10}$.

   (a) Is this model arbitrage free ?

   A. No B. Yes

   (b) Consider from now on the call option with strike $K = 150$. What is the expected value $\mathbb{E}[C_1]$ of the call ?

   A. 50 B. 45 C. 44 D. 194

   (c) What is the initial value of a portfolio which is replicating the call ? Try to solve the problem in two similar ways:

   i. Describing the portfolio as (k, h), where k the number of bonds and h of shares, whose wealth is $V_t^{k,h} := kB_t + hS_t$ for both $t = 0$ and $t = 1$.

   ii. Describing the portfolio as (x, h), where $x = k + S_0 h$ is the initial capital and h the number of shares, and whose wealth is $V_0^{x,h} := x$, $V_1^{x,h} := x(1 + r) + h(S_1 - S_0(1 + r))$.

   A. The call is not replicable B. For multiple possible values C. $\frac{50}{6}$ D. $-\frac{350}{3}$ E. $\frac{350}{3}$

   (d) Suppose that we extend the binomial market, so that one can trade not only the bond and the stock, but also the call option at (initial) price $C_0 := p$. In the extended market (B, S, C), analogously to item (c) ii, describe a portfolio as (x, h, y), where x is the initial capital, h the number of shares, and y the number of options.

   i. What random variable do I need to require to be always ≥ 0, and sometimes > 0, for the corresponding portfolio to be an arbitrage?

   A. $h(S_1 - S_0(1 + r)) + y(C_1 - p(1 + r))$.

   B. $h(S_1 - S_0(1 + r)) + y(C_1 - p)$.

   C. $x(1 + r) + h(S_1 - S_0(1 + r)) + y(C_1 - p(1 + r))$.

   D. $x(1 + r) + h(S_1 - S_0(1 + r)) + y(C_1 - p)$.

E. None of the above.

ii. If I *buy one* option, for which values of p can I make an arbitrage (by appropriately investing also in the bond and the stock) ?
A. None of the other answers B. For no value of p C. $p = \frac{50}{6}$ D. $p \neq \frac{50}{6}$ E. $p > \frac{50}{6}$
F. $p \geq \frac{50}{6}$ G. $p < \frac{50}{6}$ H. $p \leq \frac{50}{6}$

iii. If $p = 0$ and I *buy one* option, for what values of h do I get an arbitrage?
A. any $h \leq 0$ B. $h \in (0, 1)$ C. $h \in [0, 1]$ D. $h \in (-1, 0)$ E. $h \in [-1, 0]$

iv. If I *sell one* option, for which values of p can I make an arbitrage (by appropriately investing also in the bond and the stock) ?
A. None of the other answers B. For no value of p C. $p = \frac{50}{6}$ D. $p \neq \frac{50}{6}$ E. $p > \frac{50}{6}$
F. $p \geq \frac{50}{6}$ G. $p < \frac{50}{6}$ H. $p \leq \frac{50}{6}$

v. For which value(s) of $p \in \mathbb{R}$ is the extended market (B, S, C) arbitrage-free?

A. For $p = \mathbb{E}[C_1]$

B. For $p =$ the initial value of a portfolio which is replicating the call

C. For $p = \frac{350}{3}$

D. For $p = 50$

E. There is no such value of p

F. None of the above

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