This document contains 1 questions.

1. [default,O7]

   In this exercise we consider the general one-period linear market model of an arbitrage-free market (with no imperfections) i.e. assume that the final wealth of an investor is

$$V_0^{x,h} := x, \quad V_1^{x,h} := x(1+r) + h \cdot (S_1 - S_0(1+r)), \quad (x,h) \in \mathbb{R} \times \mathbb{R}^m, \tag{1}$$

   where x represent the initial capital and h the number of units of the underlying S in the portfolio, and $r > -1$ the interest rate for investing in a bank account. We will assume that $m = 1$, i.e. there is only one risky asset S, which is assumed to be strictly positive, i.e. it has value $S_1 > 0$ at time $T := 1$ and value $S_0 > 0$ at time 0. We will consider all models in the above class, i.e. we do not specify the law of the random variable S_1. Denote by $C_0(K)$ a time-zero arbitrage-free price of a call option (on S) with strike price $K > 0$.

   Consider the inequalities

   A. $(S_0 - K)^+ \leq C_0(K)$ B. $S_0 - \frac{K}{1+r} \leq C_0(K)$ C. $\left(S_0 - \frac{K}{1+r}\right)^+ \leq C_0(K)$ D. $C_0(K) \leq S_0$
   E. $C_0(K) \leq S_0 - K$

   For each inequality, consider the following questions, and choose one of the following answers:

   Questions:

   - Does the inequality hold in every model (in the given class)?
   - If so, does the inequality hold with strict inequality in every model (in the given class), or does it fail to be strict in at least some models?

   Answers:

   1. The inequality fails in some models
   2. The inequality holds in all models, but not always strictly
   3. The inequality holds strictly in all models

   In summary, answer the following questions:

   (a) What is the right answer for inequality A?
   ◯ 1 ◯ 2 ◯ 3

   (b) What is the right answer for inequality B?
   ◯ 1 ◯ 2 ◯ 3

   (c) What is the right answer for inequality C?
   ◯ 1 ◯ 2 ◯ 3

   (d) What is the right answer for inequality D?
   ◯ 1 ◯ 2 ◯ 3

   (e) What is the right answer for inequality E?
   ◯ 1 ◯ 2 ◯ 3

Hint: Although to prove that an inequality is satisfied in every model (i.e. for every choice of r, S_0, S_1, K*) you cannot just restrict to considering binomial models, to find counter-examples it is often enough to consider binomial (or trinomial) models.*

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