This document contains 2 questions.

1. [default,Q3]

   Given X, Y independent, $X, Y \sim Unif[0,1]$, use the properties of the conditional expectation to compute $\mathbb{E}((X+Y)^2|X)$.

2. [default,P7]

   Which of the following statements about the process $Y = (Y_n)_{n\in\mathbb{N}}$ are correct?

   We are assuming that $\mathbb{E}|Y_n| < \infty$ for all $n \in \mathbb{N}$. Justify carefully with either proofs or counterexamples. Recall that a random variable X is said to be bounded if $|X| \leq c$ for some constant $c < \infty$.

   (a) If Y is a martingale, then $\mathbb{E}(Y_n) = \mathbb{E}(Y_0)$ for every n.

   (b) If Y is a martingale and τ a bounded stopping time then $\mathbb{E}(Y_\tau) = \mathbb{E}(Y_0)$.

   (c) If Y satisfies $\mathbb{E}(Y_n) = \mathbb{E}(Y_0)$ for every n then Y is a martingale.

   (d) If Y satisfies $\mathbb{E}(Y_\tau) = \mathbb{E}(Y_0)$ for every bounded stopping time τ then Y is a martingale.
   Hint: Given $s < t$, $A \in \mathcal{F}_s$, $\tau := t1_{A^c} + s1_A$, compare $\mathbb{E}(Y_\tau)$ with $\mathbb{E}(Y_t)$.