Problem Set 1

1). **The Game of Find the Sweet:** In a two-player game, player A must decide whether to hide a sweet inside their left hand or their right hand. This decision is made and player A closes their fists to conceal the sweet without player B seeing. Player B then guesses which hand they think player A has hid the sweet in, choosing left or right. If player B guesses correctly, they win the sweet, otherwise player A wins the sweet.

a). Construct some reasonable payoffs for this game in all possible outcomes.

b). Draw a representation of the game in:

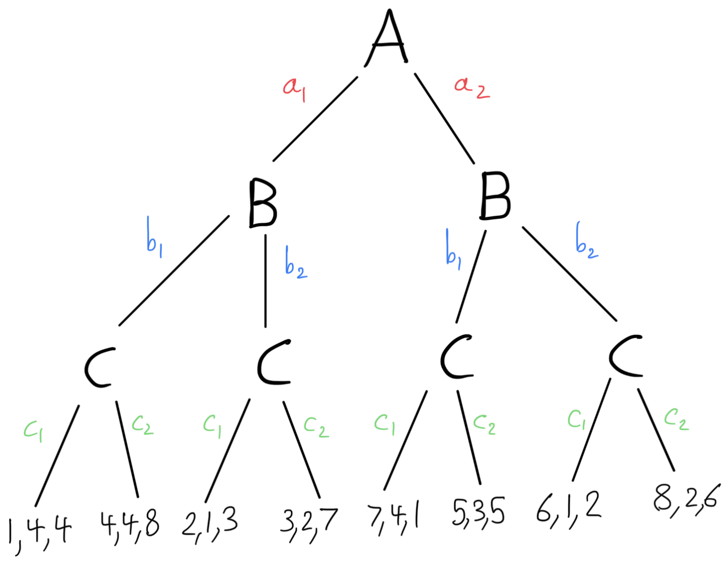
(i). Extensive form;

(ii). Normal form.

c). Comment on the pure strategies in the game, does either player have an ‘optimal’ strategy here?

d). Suppose as player B you have some knowledge of player A’s likely strategy; that more often than not, player A tends to hide the sweet in their right hand. Given this information, what should your strategy be?

2). Consider the following simultaneous move game shown in extensive form below.



a). How many players are in this game?

b). How many pure strategies does each player have?

c). Can you determine what ‘optimal’ play is in this game? Justify your choice.

3). This question is about **2 card Goofspiel**, for which we calculated each player has 4 pure strategies during lectures.

a). Describe what each of these pure strategies corresponds to in relation to the game (e.g. copy the black card).

Let's now assign some payoffs to the game in the following way. Suppose both players are not concerned by the magnitude of a win/loss, but simply care about winning or losing only. To model this let's assign a payoff of 1 for a win, 0 for a draw and -1 for a loss for each player.

b). Draw a representation of the game in normal form.

c). What do you notice about the pairs of payoffs in each cell of the normal form representation of the game? Why is this the case?

d). By removing any dominated strategies (i.e. assuming that neither player will play these), or otherwise, can you determine what 'optimal' play is in this game?

4). This question is about **3 card Goofspiel**.

a). How many pure strategies does each player have in this game?

b). The game is too large to draw out its complete normal or extensive form representation (if you have done part (a) correctly you will certainly find this a laughable understatement!), nevertheless can you intuit what 'optimal' play might be in this game? Justify your suggestion to show that it is true!

5). ($\diamond$) Note that this question is our first diamond question, meaning that the question might be open ended or a possible idea for group coursework. Usually incomplete or no solutions will be provided to these questions.

This question is about **n card Goofspiel**, where $n \geqslant 4$. Suppose that, before the start of the game, player A announces that they will play their pure strategy 'copy the black card' throughout the game, thinking that it will be optimal play. Explain why this is not the case. Acting as player B in this game, what should you do against A's strategy?

In the case when $n = 4$ can you determine what 'optimal' play is in this game?

6). ($\star$) Note that this question is our first star question, meaning that the question is considered especially difficult and often beyond the scope of our course. These are usually included for interest purposes.

This question concerns **n card Goofspiel** for any value of n. Suppose that, before the start of the game, player A announces that they will play **randomly** on every move throughout the game (i.e. at each move they will play a card from their remaining set of cards with equal probability of it being any particular one of their remaining cards). What strategy should you employ as player B against this?