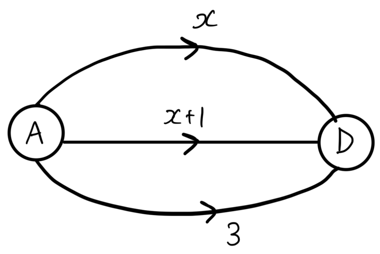
Problem Set 6

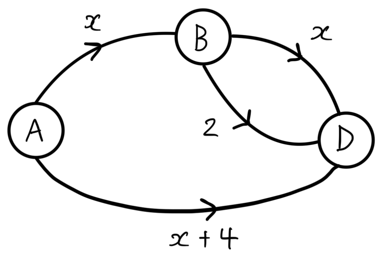
In all congestion games on this sheet, each edge shows the cost function for a flow of x users on that edge, so, for example, $x + 1$ is short for $c(x) = x + 1$.

1). In each of the congestion games labelled a) - c) below assume there are 10 users that need to get from node A to node D. For each congestion game:

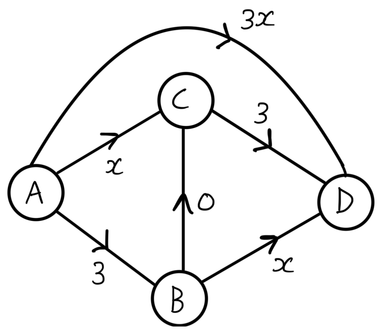
a).



b).



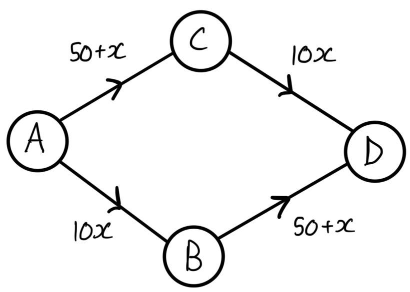
c).

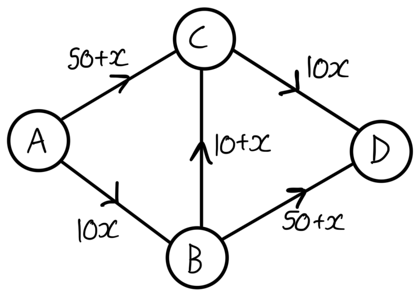


(i). Find all equilibria of the game.

(ii). Find the social optimal flow and determine the price of anarchy.

2). In the following two congestion networks suppose there are 6 users who want to travel from A to D.

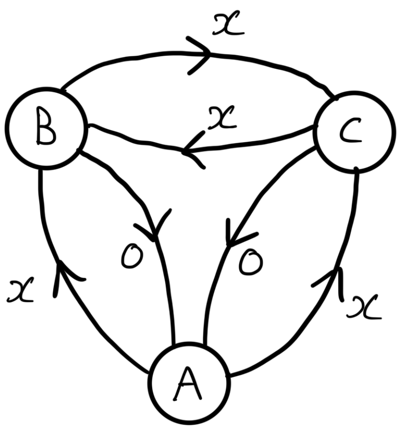




a). Find the equilibrium flow in the network on the left.

b). In the network on the right, start from the equilibrium flow in (a) and improve the path of one user at a time until you find an equilibrium flow. Compare the equilibrium costs in both networks.

3). Consider the following congestion network with three nodes. The table shows four users $i = 1, 2, 3, 4$ with different origins O_i and destinations D_i. In this network, each user has two possible routes from their origin to their destination.



User, i	O_i	D_i
1	A	B
2	A	C
3	B	C
4	C	B

Find all equilibria and the socially optimal flow in this congestion game, compare the costs of these.

4). ($\star$)($\diamond$) Prove theorem 6.37 in section 6.6 of the lecture notes for the case of splittable flow, i.e prove that for splittable flow every congestion game has at least one equilibrium.

5). ($\star$) Prove proposition 6.40 in section 6.7 of the lecture notes: for atomic flow the price of anarchy in any congestion game with affine cost functions is at most 5/2. Give an example of a congestion game with affine cost functions which results in this maximal value for the price of anarchy.

6). ($\star$) Prove proposition 6.41 in section 6.7 of the lecture notes: for splittable flow the price of anarchy in any congestion game with affine cost functions is at most 4/3. Give an example of a congestion game with affine cost functions which results in this maximal value for the price of anarchy.