

MATH60005/70005: Optimization (Autumn 23-24)

Week 10: Exercises

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1. Solve the primal and dual problem for

$$\begin{aligned} \min \quad & x_1^2 + x_2^2 + 2x_1 \\ \text{s.t.} \quad & x_1 + x_2 = 0. \end{aligned}$$

2. Study the duality gap (difference between f^* and q^*) for the problem

$$\min \left\{ e^{-x_2} : \sqrt{x_1^2 + x_2^2} - x_1 \leq 0 \right\}.$$

3. Recompute the dual of the convex quadratic problem from the notes under that assumption that the matrix $\mathbf{Q} \geq 0$ instead of $\mathbf{Q} > 0$.
4. Consider the Chebyshev center problem where we have a set of points $\mathbf{a}_1, \dots, \mathbf{a}_m \in \mathbb{R}^n$ for which we seek a point $\mathbf{x} \in \mathbb{R}^n$ that is the center of a ball of minimum radius $r > 0$ containing the points

$$\begin{aligned} \min_{\mathbf{x}, r} \quad & r \\ \text{s.t.} \quad & \|\mathbf{x} - \mathbf{a}_i\| \leq r, \quad i = 1, 2, \dots, m. \end{aligned}$$

Compute the dual of this problem. (*Hint: use an equivalent formulation over the squared radius*)

