MATH60005/70005: Optimization (Autumn 23-24)

Week 11: Exercises

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1. A simple model of a boat moving at constant speed is

$$\dot{x} = V \cos u$$
$$\dot{y} = V \sin u$$

where x and y are the positions in the xy -plane, V is the constant speed and u is the heading angle. It is desired to make a fishing trip from the initial position

$$x(0) = 0, \quad y(0) = 0$$

to the final position

$$x(1) = 1, \quad y(1) = 0$$

since there is more fish at positions with higher y -coordinates the trip is planned to maximize

$$\int_0^1 y dt$$

Show that a fishing trip satisfying the Pontryagin minimum principle has the property that

$$\tan u = c_1 + c_2 t$$

for some constants c_1 and c_2.

2. An industrial robot is configured to move a tool in one dimension. The position is x_1 and the velocity is x_2. Newton's force relation then gives the model

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u$$

where the applied force u is the control signal which is limited by

$$|u| \leq 1$$

One wishes to move the tool in such a way that it returns to its original position with maximum negative velocity, i.e. the optimization problem is

$$\min x_2(1), \text{ with } x_1(0) = 0, \quad x_2(0) = 0, \quad x_1(1) = 0$$

Compute the optimal open loop control u as a function of time.