

# MATH60005/70005: Optimization (Autumn 23-24)

## Week 2: Exercises

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1. Find the global minimum and maximum points of  $f(x_1, x_2) = x_1 + x_2$  over the unit ball in  $\mathbb{R}^2$ ,  $S = B[0, 1] = \{(x_1, x_2)^T : x_1^2 + x_2^2 \leq 1\}$ . Repeat with  $f(x_1, x_2) = 2x_1 - 3x_2$  over the set  $S = \{(x_1, x_2) : 2x_1^2 + 5x_2^2 \leq 1\}$ .
2. Classify the matrices

$$\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0.1 \end{pmatrix}.$$

3. Use a computational tool of your preference to classify

$$\begin{pmatrix} 2 & 2 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{pmatrix}, \quad \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & 2 \end{pmatrix}, \quad \begin{pmatrix} -5 & 1 & 1 \\ 1 & -7 & 1 \\ 1 & 1 & -5 \end{pmatrix}$$

4. Classify the stationary points of

- a)  $f(\mathbf{x}) = x_1^2 + x_2^2 + x_3^2 + x_1x_2 + x_1x_3 + x_2x_3 + (x_1^2 + x_2^2 + x_3^2)^2$ .
- b)  $f(x_1, x_2) = x_1^4 + 2x_1^2x_2 + x_2^2 - 4x_1^2 - 8x_1 - 8x_2$ .
- c)  $f(x_1, x_2) = x_1^2 + 4x_1x_2 + x_2^2 + x_1 - x_2$ .

