

# MATH60005/70005: Optimisation (Autumn 23-24)

## Weeks 6 and 7: Exercises

Dr Dante Kalise & Dr Estefanía Loayza-Romero  
Department of Mathematics  
Imperial College London, United Kingdom  
{dkaliseb,kloayzar}@imperial.ac.uk

1. Show the convexity of the following functions:

- the quad-over-lin function

$$f(x_1, x_2) = \frac{x_1^2}{x_2}$$

defined over  $\mathbb{R} \times \mathbb{R}_{++} = \{(x_1, x_2) : x_2 > 0\}$ .

- the generalized quad-over-lin function

$$g(\mathbf{x}) = \frac{\|\mathbf{Ax} + \mathbf{b}\|^2}{\mathbf{c}^\top \mathbf{x} + d} \quad (\mathbf{A} \in \mathbb{R}^{m \times n}, \mathbf{b} \in \mathbb{R}^m, \mathbf{c} \in \mathbb{R}^n, d \in \mathbb{R})$$

is convex over  $D = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{c}^\top \mathbf{x} + d > 0\}$ .

- $f(x_1, x_2) = -\log(x_1 x_2)$ , over  $\mathbb{R}_{++}^2$ .
- $h(\mathbf{x}) = e^{\|\mathbf{x}\|^2}$ .

2. Show that  $\sqrt{1 + \mathbf{x}^\top Q \mathbf{x}}$  is convex for  $Q$  positive definite.

3. Find the optimal solution of

$$\begin{aligned} \max_{\mathbf{x} \in \mathbb{R}^3} \quad & 2x_1^2 + x_2^2 + x_3^2 + 2x_1 - 3x_2 + 4x_3 \\ \text{subject to} \quad & x_1 + x_2 + x_3 = 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

